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AVERAGING OF THE EQUATIONS OF THE STANDARD COSMOLOGICAL MODEL OVER RAPID OSCILLATIONS

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An averaging of the equations of the standard cosmological model (SCM) is carried out. It is shown that the main contribution to the macroscopic energy density of the scalar field comes from its microscopic oscillations with the Compton period. The effective macroscopic equation of state of the oscillations of the scalar field corresponds to the nonrelativistic limit.

Keywords: cosmological model, rapid oscillations.

INTRODUCTION

Recently, in [1–3] the asymptotic properties of the standard cosmological model based on a classical massive scalar field were investigated by a combined application of methods of the qualitative theory of ordinary differential equations and their numerical integration. In particular, it was shown in these works that the system of Einstein–Klein–Gordon equations for the homogeneous, spatially flat cosmological model has one singular point, corresponding to zero values of the potential of the scalar field and its derivative, where the indicated singular point can be either an attractive center or an attractive focus or an attractive saddle. Moreover, a microscopic oscillatory character of the invariant cosmological acceleration with approach to the singular point was revealed, with an average value corresponding to the nonrelativistic equation of state. As a later study showed [4], in the case when the cosmological term of the oscillation of the scalar field and its derivative are equal to zero, although they decay with time, this process lasts for quite a long time, as much as 10^7 Compton times. In this case, the phase trajectory of the dynamical system in the $(\Phi,\dot{\Phi})$ plane practically coincides with an ideal circle, whose radius decreases slowly. This induces the thought that at sufficiently late times of the early evolution of the Universe, microscopic oscillations of the scalar field give the main contribution to the macroscopic energy density of the scalar field. In the present paper, this idea is confirmed by numerical modeling methods developed in [5].

1. MAIN RELATIONS FOR THE STANDARD COSMOLOGICAL MODEL

The energy-momentum tensor of the classical scalar field has the form

$$T^{ik} = \frac{1}{8\pi}\left(2\Phi^{,i}\Phi^{,k} - g^{ik}\Phi_{,j}\Phi^{,j} + g^{ik}m^2\Phi^2\right). \tag{1}$$

Setting the covariant divergence of this tensor equal to zero leads to the equation of the free classical scalar field:

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